GN47: Stochastic Modelling of Economic Risks in Life Insurance

Classification
Recommended Practice

MEMBERS ARE REMINDED THAT THEY MUST ALWAYS COMPLY WITH THE PROFESSIONAL CONDUCT STANDARDS (PCS) AND THAT GUIDANCE NOTES IMPOSE ADDITIONAL REQUIREMENTS UNDER SPECIFIC CIRCUMSTANCES

Definitions
Defined terms appear in italics when used in the standard.

Purpose

The purpose of this Guidance Note is to provide specific guidance on the use of techniques for stochastic modelling of economic risks within a life insurance *firm* with particular reference to its regulatory obligations. The guidance provides a common framework for assessing the robustness of the models and calibration approaches used in stochastic modelling and the uses to which the modelling is put. The guidance is not intended to constrain the further development of stochastic modelling techniques.

Reference	**Definition**
Asset model	A model used to generate a set of stochastic scenarios for a particular asset class
Economic Scenario Generator (ESG)	A model which combines the results from one or more *asset models* to produce a consistent set of multi-asset scenarios.
firm	The life insurance firm in respect of which stochastic modelling is being used in relation to reserving and capital assessment
FSA	Financial Services Authority
Individual Capital Assessment (“ICA”)	The assessment required by PRU 1.2.26R of the capital which a *firm* needs to hold to meet PRU 1.2.22R (adequate financial resources, including capital resources)
moneyness	the degree to which an option is in or out of the money
Volatility Surface	The variation of implied volatility with features of an option, e.g. *moneyness* and term for options term and tenor for swaptions

The following terms have the same meaning as in the *FSA* Handbook of Rules and Guidance:

Long-term insurance business
Principles and Practices of Financial Management ("PPFM")
risk capital margin
with-profits business
With-profits insurance capital component ("WPICC")

Legislation or Authority
The Financial Services and Markets Act 2000
The *FSA* Handbook of Rules and Guidance: Integrated Prudential sourcebook ("PRU")
The *FSA* Handbook of Rules and Guidance: Interim Prudential sourcebook for insurers ("IPRU (INS)")

Application
*Firm*s using stochastic modelling of economic risks when reserving for options and guarantees in life insurance policies or assessing the amount of capital required to support *long-term insurance business*.

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Status
Approved under Due Process

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1. General

1.2 This Guidance Note is drafted in terms which are not addressed to actuaries specifically. Nevertheless, actuaries performing work covered by this Guidance Note are required to apply it according to its classification. However, where a *firm* requires an actuary to produce work conflicting with this Guidance Note, the actuary may do so provided the work clearly and unambiguously states that the actuary has done so under instructions and that the work does not conform to this Guidance Note.

1.3 If the development of stochastic modelling of economic risks within a *firm* is such that one or more material aspects of this Guidance Note are not being complied with, the extent of non-compliance and the alternative adopted should be recorded in the report of the valuation or capital assessment to which it refers. There may be other practices not set out in this note that constitute generally accepted actuarial practice in this area and failure to comply with this note does not necessarily imply failure to follow generally accepted actuarial practice. It is recognised that stochastic modelling is a developing area of practice and *firms* will need to consider the extent to which plans should be put in place to continue development of stochastic modelling, with particular consideration being given to how all material aspects of this Guidance Note, or justified equivalent alternatives, could be met.

1.4 This guidance note provides guidance on the use of stochastic modelling in the context of PRU 7.3 (Mathematical Reserves), PRU 7.4 (*With-profits Insurance Capital Component (WPICC)*) and PRU 2.3 (*Individual Capital Assessment (ICA)*). It also includes some summarised references to, or quotations from, particular provisions of the *FSA* Handbook of Rules and Guidance (the "*FSA* Handbook"), but users should be aware that this is not exhaustive and does not provide a substitute for referring to the *FSA* Handbook.

1.5 This Guidance Note should be read in conjunction with GNs 44 – 46, which contain guidance on the circumstances in which the use of stochastic modelling is favoured by *FSA* and on some aspects of the way in which modelling should be applied if used.

1.6 Stochastic models are likely to be used in two distinct ways for the purposes covered by this standard. The first is to obtain a 'market-consistent' value of a liability ('market-consistent' is defined in paragraph 3.1 below). The second is to establish an amount of assets that will enable the *firm* to meet its liabilities to a desired probability level.

1.7 The types and/or parameterisation of stochastic modelling which it is appropriate to use may differ according to the purpose of the calculation (e.g. valuation or capital assessment) or the nature of the guarantee (e.g. minimum return from equity portfolio or guaranteed annuity rate).

2. Algorithms for Computing Market-Consistent Values

2.1 Given a market-consistent *asset model* and a liability description, there may be several possible methods for computing the market-consistent liability valuation. The possibilities include in particular:

- closed form modelling; and
- Monte Carlo simulation.

2.2 Guidance in the remainder of this note has been prepared in the context of the closed form modelling and Monte Carlo simulation methods referred to above although some is of more general application.

2.3 Closed form Modelling

2.3.1 Models should be arbitrage free, but for certain options, it may be possible to use the formula underlying the model structure selected to derive market-consistent values. However, it is necessary to ensure that the formula used adequately reflects both management and policyholder actions, unless these are very limited in possible effect or their omission can be shown to be appropriately prudent.

2.4 Monte Carlo Simulation

2.4.1 It is equally appropriate to use a risk-neutral probability measure, discounting at risk-free rates, or any other measure (including ‘real world’ measures), discounting using consistent deflators. It is appropriate either to generate independent equally likely simulations or to adopt variance reduction techniques in relation to the model or both.

2.4.2 The sampling errors involved in Monte Carlo simulation should normally be estimated. This may be by

a) using analytical formulae for standard errors, or

b) using increasing numbers of simulations until a number can be identified beyond which additional simulations add little additional accuracy to the valuation, or

c) in some other way.

If the sufficient number of simulations approach is used, it is not necessary to use the upper bound of a confidence interval for the estimate of the value.

2.4.3 The pseudo random number generator underlying the model(s) should have been tested to ensure that it produces numbers which display sufficient randomness.

3. *WPICC* ‘Market-Consistency’ in the context of PRU 7.4

3.1 PRU 7.4.169R(1) requires any stochastic approach used for valuing guarantees, options and smoothing when calculating *WPICCs* to be ‘market-consistent’. PRU 7.4.170R defines this as “… a model that delivers prices for assets and liabilities that can be directly verified from the market …” and requires the model to be calibrated “… to deliver market-consistent prices for those assets that reflect the nature and term of the *with-profits insurance liabilities*”. Additional guidance is given in PRU 7.4.176-180G.

3.2 Underlying model structures should be arbitrage free. However to the extent that there are small arbitrage opportunities in the simulations produced, this is acceptable provided they are not exploited for the benefit of the results.

3.3 In the context of *with-profits business,* assets “reflecting the nature and term” of the liabilities would include those assets the return on which is used to determine policy payouts (i.e. those deemed to constitute the asset shares of the policies being valued). It would also include derivatives, particularly ‘European’ put options on the assets constituting, or reasonably close to those constituting, the asset share if policies contain guaranteed minimum maturity values and interest rate swaptions if guaranteed annuity rates are being valued. The model used should also be capable of reproducing the prices of differently credit-rated stocks (other than those excluded by PRU 7.4.87R), if such stocks form a material part of asset shares.

3.4 Except where the contrary is expressly stated in PRU, the basis for the valuation of assets and liabilities is set out in PRU 1.3.5R. In particular, this does not apply to calculation of the mathematical reserves. ‘Market-consistent’ values should be interpreted consistently with this rule. In particular, if the rule requires the use of bid or offer prices rather than mid-market prices, then input parameters or output values should be adjusted to produce the appropriate results, including an estimate of the spreads which market-makers may apply to large, infrequently traded over-the-counter instruments. Unless otherwise directed or implied by *r*ules and guidance in the *FSA* Handbook, it is not necessary to assume that the expected ‘close-out’ cost of an unhedged position after a very short term market shock will be subject to wider than normal price spreads.

3.5 Where no established derivative market exists, for example options on commercial property, it is impossible to base the calibration on the market, and the use of historical data for calibration may be a suitable alternative.

3.6 Where data are based upon surveyors’ valuations, as is typical in many property market indices, historical performance may contain significant elements of smoothing relative to similar, although sparser, observations from realised sale prices. It is more appropriate to base the implied volatility for a market-consistent valuation of guarantees on the price at which an asset can be realised. Where only surveyors’ valuations are available, appropriate adjustments should therefore be made to the dataset, to the parameters or to the results in order to remove the possible impact of smoothing. Parameters based on surveyors’ valuations may be used alongside parameters based upon sale prices for circumstances where this better reflects current practice. Examples include the calculation of asset share, and the effect of management actions contingent on the value of assets, where the practice of the *firm* may be to use surveyors’ valuations.

3.7 Model calibration

3.7.1 The model used should be one that has been shown to reproduce option features (either prices or volatilities) as at the valuation date sufficiently accurately, subject to paragraph 3.10 below. Option prices for the underlying investments should be reproduced taking into account the relevant *volatility surface* or vice versa. The option features reproduced should be for options where no significant credit risk is taken on. However, in accordance with paragraph 1.2 above, approximate methods may be used. Any such approximations should be expected to be of overall neutral effect and their use should be disclosed in accordance with paragraph 1.2 above.

3.7.2 Approximations might include using different parameterisations of a constant volatility model to value different model points (in which case the impact of aggregate level management actions needs to be allowed for) or using a single parameterisation calibrated to a point which is an appropriately weighted average of points on the *volatility surface.*

3.7.3 If differently parameterised models are used for different sets of liability model points, then close replication would only be required for options corresponding to the model points to which the parameters are applied. Consistent modelling of management actions between model point sets is necessary.

3.8 Unavailability of data

3.8.1 In many situations, options may be infrequently traded, or price data may not be available for options of strike, term or credit quality corresponding to the liabilities. In this situation it is acceptable to calibrate a model to the longest available price data, or the closest available *moneyness*, or the nearest available credit quality of issuer. This parameterisation of the model should then be extrapolated to the term, *moneyness* or desired credit quality of the calibration.

3.8.2 Extrapolation should allow for the continuation of any observed trend and, unless there is no trend, unchanging parameters should not be assumed as term, *moneyness* or credit quality become more extreme. On occasions, it may be most appropriate to extrapolate along a curve with a turning point if justified by recent market price observation or underlying economic theory. Any choice between alternative parameters for extrapolation should be justified.

3.8.3 Where the longest quoted prices may themselves have been extrapolated or are not based on recent trades then the adequacy of the extrapolation relative to the preceding guidance should be considered and adjusted if non-compliant.

3.8.4 If option prices are not available on a particular index at certain durations but are available on an index which may share some similar characteristics, an appropriately adapted parameterisation to that second index may be used.

3.8.5 For any asset class, calibration should be adjusted to allow for changes in the volatility where there is bias relative to the index used (e.g. a territorial bias relative to an international index or a sector bias relative to a national index) or where there are individual large holdings.

3.8.6 If over the counter hedging assets are held by the *firm* then these can be used in the calibration exercise if they closely match the liabilities. The calibration to such assets should be performed recognising the guidance given in section 3.7 above.

3.9 Where more than one asset class is being modelled stochastically it is necessary to make assumptions about dependencies between the different asset classes (this may include dependencies between asset classes in different economies). Where implied correlations cannot be obtained due to the absence of a deep and liquid market in this type of instrument it is appropriate to calibrate a model to correlations based on historic averages. Historic averages should be adjusted to reflect available data on implied correlation spreads or economic reasoning with any such adjustment being justified. If a *firm* holds options hedging a guarantee which imply certain correlations, then these correlations may be used to calibrate the model provided the options hedge a significant proportion of the guarantees valued by the model.

3.10 Where the definition of the 'risk-free rate' parameter or 'risk-free curve' for the valuation of the insurance liabilities (see in particular paragraph 4.1.3 of GN45) differs from that implicit in the market price of otherwise relevant options, it may no longer be possible to demonstrate market consistency by direct comparison between the observable market values of particular assets and the values generated for the same options by the liability valuation approach. In such cases a two stage approach to the demonstration of market consistency may be appropriate. In the first stage relatively simple closed form solutions may be parameterised to match the market value of observable options using a consistent discount rate, frequently the swap rate. These closed form solutions and the same parameters should then be reused with the discount rate adjusted to match the selected risk-free rate or curve to establish theoretical market values consistent with the definition of risk-free used in the valuation of the liabilities. These theoretical market values can then be used to validate the market consistency of the liability valuation approach by confirming that the liability approach adequately reproduces those theoretical market values. Alternative approaches such as the calibration of two scenario files (one using a market practice based definition of the discount rate, such as swap rates, and the other maintaining all parameters, but replacing the discount rate with the selected risk-free definition) may be used, but regard should be had as to whether the level of transparency given by such approaches is sufficiently high.

3.11 IPRU(INS) Appendix 9.4A 6(4)(a)(iii) requires the completion of a table of values of specified assets as calculated by the stochastic model used for the purposes of PRU 7.4. This should be done using the same parameterisation of a closed-form model or the same set of simulations for a Monte Carlo approach used to value the liabilities, even though a model calibrated to a risk-free rate (as defined in GN45) will not exactly reproduce market-observable prices for the specified assets.

4. Practical use of Market-Consistent Models in the calculation of the *WPICC*

4.1 Maturity Guarantees

4.1.1 For a policy under which the maturity benefit is the larger of a quantity related in some way to the value of underlying assets and a guaranteed amount (which may increase in future as bonuses are added), 'assets that reflect the nature and term of the liabilities' (PRU 7.4.170R) should include appropriate put or call options on the underlying assets.

4.1.2 To the extent that the underlying assets are equities which are invested broadly in line with a recognised index, the model used should normally replicate put option prices on that index at durations and strikes appropriate to the term of the guarantees**.** The model used should, however, reflect the receipt of dividends as well as capital growth and allow for the impact of tax at an appropriate rate.

4.1.3 If fixed interest assets are pooled (i.e. an identical return is attributed to all policies independently of term), then stochastic modelling of the returns would be appropriate, using a model which is capable of reproducing swaption prices for a range of exercise dates corresponding to the range of policy guarantee dates and tenors (lengths of swaps) corresponding to the outstanding terms of the assets intended to be held at each guarantee date. Allowance should be made for the guidance in paragraph 3.10 above when determining the swaption prices.

4.1.4 Even if differential returns from fixed interest assets are allocated to asset shares, exact matching cannot be achieved in both the base scenario and the persistency stress scenario and the exposure to market risk in respect of assets matching liabilities subject to persistency stress should be allowed for. For other assets, approximate methods may be appropriate for any residual risk. Allowance should be made for the guidance in paragraph 3.10 above.

4.1.5 The stochastic modelling of fixed interest assets discussed in paragraphs 4.1.3 and 4.1.4 above should allow for any credit risk in the assets held.

4.2 *Risk Capital Margin*

4.2.1 In the calculation of the *risk capital margin*, it is necessary to revalue liabilities in scenarios of changed prices.

4.2.2 It is also necessary to assume that all assets (including hedging assets (e.g. equity put options or interest rate swaptions)) are revalued in line with the changed prices, ensuring that allowances for issuer credit risk are preserved (or, in the case of credit stress, appropriately adjusted).

4.3 Reserves for Smoothing etc

4.3.1 A reserve (or possibly an asset) in respect of smoothing is an element of the realistic balance sheet. This should normally be calculated stochastically.

4.3.2 *FSA* encourages in PRU 7.4.178G an holistic approach to stochastic modelling. Other items on the realistic balance sheet which may be calculated stochastically include inflation in expenses (because expenses may impact upon guarantee or option costs), profits or losses from early terminations, regular or terminal charges against or credits to asset share, misselling compensation and investment expenses.

4.3.3 The value of future profits from non-profits business should be calculated on a stochastic basis, in respect of market risk, if there is material exposure to guarantees or options (unless fully hedged) inherent in the projected profit stream.

5. Use of Stochastic Models in *Individual Capital Assessment*

5.1 This section provides advice on the design and calibration of a stochastic model for use in quantifying the capital requirement in relation to economic risks. For the avoidance of doubt, this section does not apply to the use of stochastic models for the calculation of market-consistent values of liabilities for the purposes of *ICA* calculations pursuant to PRU 2.3.14G.

5.2 GN46 - Individual Capital Assessment contains guidance on calculating an *ICA*, including the use of results from a stochastic model.

5.3 Choice of Model and Parameters

5.3.1 The *ICA* framework chosen by the *firm* will normally specify the probability levels and their associated time horizons to be used. The choice of model and its parameters should be made with these probability levels and time horizons in mind.

5.3.2 It is necessary to ensure that the probability distribution used can properly reproduce the more extreme historically observed behaviour of the variable being modelled both in the size of the tail of the distribution being modelled and, where appropriate, in the path taken during the simulation period.

5.3.3 It should be recognised that there will be limited historical observations of the more extreme tail outcomes, even for the most common economic variables. A considerable degree of uncertainty will therefore exist in the behaviour of the tails of distributions.

5.3.4 The method used to combine distributions of different variables to arrive at a combined model to enable the determination of the amount of capital which satisfies the level derived in paragraph 5.3.1 is of key importance. For example, there may be a stronger observed or anticipated relationship between variables in more extreme stress scenarios. If this is the case, consideration should be given as to whether simple combination approaches (i.e. involving a fixed correlation) or assumptions of independence are adequate.

5.3.5 In the context of an *ICA*, it is necessary to use models for all asset classes which reflect real-world parameters. It is also desirable that models are arbitrage free. To the extent that arbitrage opportunities do exist in the model they should not be exploited to reduce the required capital.

5.3.6 To obtain 'real world' outcomes, it is generally appropriate to calibrate models with reference to actual historic parameters. Maximum Likelihood Estimation ('MLE') may be appropriate in some circumstances. However, particularly where fit to the tail of a distribution is more important than the overall fit, alternative techniques such as quantile matching may be more appropriate.

5.3.7 The length of the period covered by the historical data that is relevant may be limited by availability (e.g. the UK property market). In other cases, data may be available going back much longer (e.g. UK gilt yields). An assessment should be made of the data available and the effect that different lengths of observation period would have. The selected parameter should also be consistent with the *firm*'s underlying future economic expectations.

5.3.8 Similarly, correlations between variables should be calculated over longer and shorter periods and the results compared. To the extent that there have been material changes in level between different time periods, correlations should be selected consistent with the *firm's* underlying future economic expectations.

5.3.9 The choice of data used to parameterise a model should be appropriate for the purpose. There may be circumstances in which a model fits the available data but is not appropriate for use in an *ICA* for the reasons set out in paragraphs 5.3.2-5.3.4. Similarly adjustments may be required to reflect any differences between the investments included in the data series use for parameterisation and the actual investments held. Current market conditions and expert judgement may also be sources of relevant information in fitting a model and choosing parameters.

5.4 Modelling Specific Economic Risks

5.4.1 It is common for *asset models* to be combined into an *ESG* to produce more than one variable.

5.4.2 Equities

5.4.2.1 There is no explicit restriction on the choice of equity models; in particular, mean-reverting models may be used and no maximum equity risk premium is prescribed. If a lognormal or other simple model cannot properly reproduce the more extreme historically observed behaviour adequately, either a model exhibiting more appropriate skewness and kurtosis should be used or the simpler model should be used with adjusted parameters to provide sufficient outcomes in the relevant tail.

5.4.3 Interest Rates

5.4.3.1 Most published interest rate models are designed to calculate market-consistent financial instrument prices under risk neutral assumptions. When using such models for 'real world' projections, care should be taken to ensure that the results are appropriate. In particular, the distributions and volatilities of interest rates at different future points in time should be checked to ensure that any trends are plausible.

5.4.4 Corporate bonds

5.4.4.1 For fixed-interest stocks (other than those stocks issued or guaranteed by AAA rated governments like most EU governments and the US Treasury where the market appears to price the bonds as if they are substantially risk-free) then it is likely to be necessary to model variation in prices and default rates/recoveries. Where possible, models should be calibrated to historic spread variation, re-rating, default and recovery experiences. The additional risks associated with any lack of diversity should also be modelled.

5.4.5 Credit risks should be modelled for all fixed interest, index linked and floating rate stocks, not just those backing with-profits liabilities (except for stocks not allocated to back any liability or capital requirement).

5.4.6 Both market risk and credit risk should ideally be incorporated in the same model if non-governmental stocks form a material part of the assets. Where this is not possible, the capital requirements should normally be modelled separately and combined using an appropriate aggregation technique. Either method should normally allow for appropriate correlations to be incorporated between adverse credit and equity scenarios.

5.4.7 Inflation

5.4.7.1 It will generally be appropriate to model inflation stochastically, especially where significant exposure exists to administration expenses recovered from charges which are not linked to a prices index (e.g. as a percentage of funds under management). The relevant income from charges should be modelled using consistent stochastic assumptions.

5.4.7.2 Inflation risk should also be modelled stochastically if a material quantity of prices-index linked policy liabilities exist and adequate close matching assets are not held.

5.5 Benchmarks for Calibrations (*ICA*)

5.5.1 Market-consistent models can be calibrated against the market prices of assets that reflect the nature and term of the liabilities. No such calibration benchmark exists for the 'real world' models required to calculate an *ICA*. This section gives guidance against which certain aspects of some models can be tested.

5.5.2 The data available on the distribution of asset returns is sparse. Benchmarks for distribution can be constructed, for example by fitting a distribution to available historic UK data. There is, however, some evidence that the data exhibits heavier tails than are given by many distributions. Therefore, models based on such fitted distributions tend to understate the frequency of more extreme outcomes.

5.5.3 When modelling less diversified portfolios or individual equities it may be necessary to increase the modelled variation to allow for idiosyncratic risk.

6 Stochastic Modelling for Mathematical Reserves

6.1 PRU 7.3.67G states that "where there is considerable variation in the cost of the option depending on the conditions at the time the option is exercised and where that variation constitutes a material risk for the *firm*, it will generally be appropriate to use stochastic modelling".

6.2 Further, PRU 7.3.68G states that: "where the option offers a choice between two non-discretionary financial benefits (such as between a guaranteed cash sum or a guarantee annuity value, or between a unit value and a maturity guarantee) and where there is a wide range of possible outcomes, the *firm* should normally model such liabilities stochastically." This requirement applies to all such forms of liability including those from *with-profits business* and unit linked business.

6.3 PRU 7.3.67G comments that prices from the *asset model* used in the stochastic approach should be benchmarked to relevant market asset prices before determining the value of the option. However, PRU 7.3.10R(4) requires the inclusion of "… appropriate margins for adverse deviation of relevant factors". PRU 7.3.16G comments that "the margins for adverse deviation of a risk should generally be greater than or equal to the relevant market price for that risk".

6.4 Consequently, a value produced by a model that satisfies the guidance given in sections 2,3 and 4 above for calculating market consistent values represents a lower bound for the relevant mathematical reserve.

6.5 In determining the appropriate mathematical reserve, further margins for adverse deviation may be allowed for by:

- adjusting the individual assumptions relevant to the price of the option, (these adjustments may include but not be limited to, assumptions on yield curve, implied volatilities or mortality tables); or
- adding a single overall margin for adverse deviation to the derived option price.